

Gravitational Waves from the Non-Perturbative Decay of Condensates along Supersymmetric Flat Directions

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It has recently been shown that specific non-perturbative effects may lead to an explosive decay of flat direction condensates in supersymmetric theories. We confirm explicitly the efficiency of this process with lattice simulations: after few rotations of the condensates in their complex plane, most of their energy is quickly converted into inhomogeneous fluctuations. We then point out that this generates a gravitational wave background which depends on the inflaton sector and falls in the Hz-kHz frequency range today. We compute the resulting spectrum and study how it depends on the parameters. We show that these gravity waves can be observable by upcoming experiments like Advanced LIGO and depend crucially on (i) the initial VEV of flat directions when they start to oscillate, (ii) their soft SUSY-breaking mass and (iii) the reheat temperature of the universe. This signal could open a new observational window on inflation and low-energy supersymmetry.

Gravity wave (GW) experiments could provide unique informations about high-energy phenomena in the early universe, such as inflation, preheating after inflation, cosmic strings and first order phase transitions [1]. In particular, cosmological GW backgrounds will be a target for several high-sensitivity interferometric experiments, planned or proposed, in the frequency range between 10^{-5} and 10^3 Hz. In this letter, we want to consider a new cosmological source of GW, which emerges naturally in the framework of high-energy physics.

Supersymmetric theories typically involve many flat directions (see [2] for the MSSM), i.e. directions in field space where the renormalizable part of the scalar potential is exactly flat in the limit of unbroken supersymmetry. The flatness is lifted by soft SUSY-breaking terms, non-renormalizable terms and SUSY-breaking terms from the finite energy density in the early universe [3]. Scalar field condensates may develop large VEVs along these directions during inflation and have several interesting consequences in cosmology [4]. A major example is the Affleck-Dine mechanism for baryogenesis [5]. Condensates with large VEVs start to oscillate when the Hubble rate H becomes of the order of their soft mass $m \sim \text{TeV}$ [6], with an initial amplitude that is model-dependent and could be as high as the Planck scale [3, 8]. It has recently been shown [9, 10, 11], that non-perturbative resonant effects may lead to an explosive decay of these coherent oscillations. We will show that this generates GW that can be observable by upcoming experiments [12].

The resonant decay of flat directions shares many similarities with preheating after inflation, when the inflaton decays into large, non-thermal fluctuations of itself and other bosonic fields [14]. GW from preheating have been intensively studied recently [15, 16, 17, 18]. Ref. [15] developed theoretical and numerical methods to calculate GW production from dynamical scalar fields in an expanding universe. We will apply and extend these methods to the decay of flat directions.

The resulting GW, however, will have different properties. GW from preheating have been studied so far in

two main classes of models: parametric resonance after chaotic inflation and tachyonic preheating after hybrid inflation. In the first case, the typical GW frequency today is of the order of $f \sim 10^7$ Hz, which is too high to be observable. Hybrid inflation models may occur at lower energy scales, so it was initially hoped that GW from tachyonic preheating may fall into an observable range. However, as was shown in [16], this generically requires very small coupling constants [19]. By contrast, we will see that GW from the non-perturbative decay of flat directions fall naturally in the Hz-kHz frequency range today, where high-sensitivity interferometric experiments can operate. Another difference - which will be crucial for the GW amplitude - is that, whereas the inflaton dominates the energy density before it decays, flat directions are usually subdominant (at least for sub-Planckian VEVs and when their non-perturbative decay is sufficiently fast). During their decay, the universe can then be matter-dominated if the inflaton is still oscillating around the (quadratic) minimum of its potential, or radiation-dominated if it decays earlier and its decay products thermalize when $H > m$. Each case leads to different GW spectra, so GW from the non-perturbative decay of flat directions carry also informations on the inflaton sector.

To illustrate why it may be difficult for GW from preheating to be observable, consider a simple model, $V = m^2\phi^2 + g^2\phi^2\chi^2$, with two *real* scalar fields ϕ and χ . In this model, χ -particles are produced due to the non-adiabatic evolution of their frequency as the condensate ϕ oscillates around the minimum [14]. For $m \sim \text{TeV}$, this could *a priori* describe the decay of a flat direction ϕ . We will denote by an index "i" the time t_i when the flat direction starts to oscillate ($H_i = m$), by Φ_i its initial amplitude at that time and we normalize the scale factor to $a_i = 1$. The spectra of the quanta amplified by preheating are usually strongly peaked around some typical (comoving) momentum k_* , which is inherited by the GW spectrum [15, 16, 18]. For the model above, $k_* \sim \sqrt{gm}\Phi_i a^{1/4}$ [14]. Assuming for simplicity that the universe is already radiation-dominated during the de-

cay, this leads to the estimate $f_* \sim \sqrt{g\Phi_i/M_{\text{Pl}}} 10^{11}$ Hz for the peak frequency of the resulting GW today. Taking for instance $\Phi = 10^{-2} M_{\text{Pl}}$, $f_* < 10^3$ Hz requires an extremely small coupling constant $g^2 < 10^{-28}$. If Φ_i is smaller, the GW amplitude is too low to be observable. Indeed, since the energy density in the flat direction ρ_{flat} is subdominant, only a small fraction $\rho_{\text{flat}}/\rho_{\text{tot}}$ of the total energy density is available for the production of GW. We will see that the fraction of energy density in GW varies as $(\rho_{\text{flat}}/\rho_{\text{tot}})^2$, which can be very small.

However, flat directions are complex fields and non-renormalizable terms usually generate a velocity for their phase when $H \sim m$ [3, 5]. The subsequent dynamics is dominated by the $m^2|\phi|^2$ term, resulting in a constant, non-zero phase velocity and an elliptical motion for $a^{3/2}\phi$ in its complex plane. This leads to time-dependent (and quasi-periodic) mixings between different excitations around flat directions, which may result in a very efficient mechanism of particle production [9, 10], significantly different from the model considered above. In particular, these particles have a typical momentum $k_* \sim m$, of the order of the Hubble scale. Formally, this amounts to replace $\sqrt{g\Phi_i}$ by \sqrt{m} in the previous model. This gives a peak frequency of GW that is independent of Φ_i and of order 10^3 Hz today for $m \sim \text{TeV}$.

This encouraging estimate should be complemented by detailed computations. The resonant decay of flat directions has been studied so far only in the linear regime, neglecting any backreaction of the quanta produced. Backreaction is eventually responsible for most of the decay and we have to use lattice simulations to fully take it into account. We will simulate the model [9, 20]

$$V = m^2 |\phi|^2 + m_\chi^2 |\chi|^2 + \frac{g^2}{2} (\phi \chi^* + \phi^* \chi) \quad (1)$$

where $*$ denotes the complex conjugate, ϕ the flat direction field and χ its decay products. This potential includes the relevant terms for excitations around MSSM flat directions (where the interaction comes from the D-term) but at least two flat directions have to be considered when the symmetry broken by their VEV is gauged [9]. This case was studied in [10, 11] and found to be largely similar to the model (1), so here we focus on (1) for simplicity. We perform lattice simulations with the parallel code ClusterEasy [21] and we compute the resulting GW spectrum with the method of [16], extended here to take into account the expansion of the universe.

Consider first the non-perturbative decay itself. The background evolution of the condensate is given by $\phi = (\cos mt + i \epsilon \sin mt) \Phi_i / \sqrt{2a^3}$. The "ellipticity" $\epsilon \leq 1$ is model-dependent but typically of order $\epsilon \sim 0.1$ [3]. An analytical study of particle production in this background will appear elsewhere. Here we just present the results. It is convenient to introduce the parameter $q = q_i/a^3$ with $q_i = g^2 \Phi_i^2 / m^2$. Note that q_i can be as high as 10^{30} for Φ_i close to the Planck scale. In the realistic case $q\epsilon^4 \gg 1$ [22], particle production occurs because the physical eigenstates vary with time [9]. Fig. 1 shows

the spectra of the χ -modes in a Minkowski background when backreaction is still negligible. The resonance is characterized by large instability bands separated by narrow stability bands, contrary to parametric resonance but similarly to the tachyonic resonance studied in the second ref. of [23]. When the trajectory of the condensate in its complex plane is circular, $\epsilon = 1$, there is a tachyonic amplification of the modes with $k^2 + m_\chi^2 < m^2$ [9]. When $\epsilon < 1$, the amplification is more efficient and extra instability bands appear in the UV, roughly up to $k < m/\epsilon$. The IR band has the highest amplitude, except if m_χ is very close to m . The peak of the spectrum is located at $\kappa_* \equiv k_*/m \sim 0.1$. In an expanding universe, the physical momenta k/a "move" in the resonance pattern and the peak is shifted to a *comoving* momentum $\kappa_* \sim 1$. When the χ -modes have been sufficiently amplified, they backreact on the condensate and convert its energy into inhomogeneous fluctuations of both fields. This will be responsible for most of the GW production. We show the evolution of the mean of $a^{3/2}\phi$ in its complex plane in Fig. 2. The field first follows its elliptical trajectory, but the coherent motion is then quickly destroyed by backreaction. This typically happens after only 1 to 5 rotations of the condensate (the higher g^2 , the sooner it occurs).

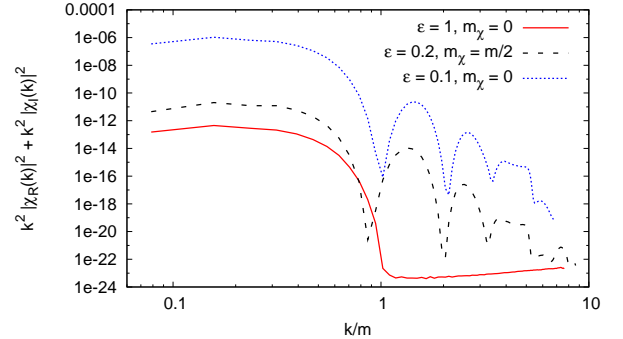


FIG. 1: Spectra of the χ modes without expansion of the universe when backreaction is still negligible. They are independent of q as far as $q\epsilon^4 \gg 1$. For $q\epsilon^4 \ll 1$, particle production is much less efficient.

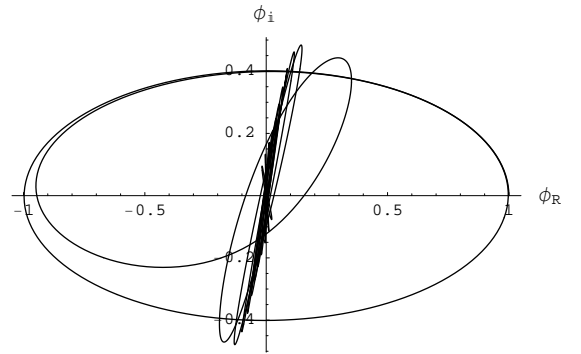


FIG. 2: Evolution of the mean of the real and imaginary parts of $a^{3/2}\phi$ in a matter-dominated background with $\epsilon = 0.4$, $g^2 = 0.01$, $m = 1$ TeV and $\Phi_i = 10^{-12} M_{\text{Pl}}$.

We now turn to the actual computation of GW. Their

evolution from the time of production (that we will denote with an index "p") up to now depends on the background equation of state, see e.g. [15] for details. In our case, their present-day frequency and energy density per logarithmic frequency interval [1] read

$$f = \frac{k}{\rho_i^{1/4}} \left(\frac{a_i}{a_{RD}} \right)^{1/4} 4 \times 10^{10} \text{ Hz}$$

$$h^2 \Omega_{\text{gw}} = 9.3 \times 10^{-6} \frac{1}{\rho_i} \left(\frac{a^4}{a_i^4} \frac{d\rho_{\text{gw}}}{d \ln k} \right)_p \left(\frac{a_i}{a_{RD}} \right) \quad (2)$$

where k is the comoving wave-number and ρ_i is the *total* energy density at t_i . If the universe is matter-dominated (MD) during the flat direction decay and becomes radiation-dominated (RD) later, when $a = a_{RD}$, then GW are further redshifted and diluted by the factors in a_i/a_{RD} above. These factors are absent if the universe is already RD at t_i [24].

We can estimate how the GW spectrum depends on the parameters from the characteristic physical length $R_* = a/k_*$ of the scalar field inhomogeneities [15, 16, 18], but we have to take into account that the flat direction energy density ρ_{flat} is usually subdominant. We then estimate the fraction of energy density in GW at the time of production as [25] $\rho_{\text{gw}}/\rho_{\text{tot}} \sim 0.1 (R_* H)^2 (\rho_{\text{flat}}/\rho_{\text{tot}})^2$. Using $\rho_{\text{flat}i} = m^2 \Phi_i^2$, $k_* = \kappa_* m$ and $H_i = m$, Eqs.(2) give

$$f_* \simeq \kappa_* \sqrt{\frac{m}{\text{TeV}}} \left(\frac{a_i}{a_{RD}} \right)^{1/4} 5 \times 10^2 \text{ Hz} \quad (3)$$

$$h^2 \Omega_{\text{gw}}^* \sim \frac{10^{-4}}{\kappa_*^2} \left(\frac{\Phi_i}{M_{\text{Pl}}} \right)^4 \left(\frac{a_i}{a_{RD}} \right) \quad (4)$$

for the frequency and amplitude of the GW spectrum's peak. Note that the amplitude is very sensitive to Φ_i . Note also that Eq.(4) cannot be extrapolated to arbitrary large Φ_i . If a flat direction acquires an initial VEV of order $M_{\text{Pl}}/3$ or larger, it dominates the total energy density before decaying and its oscillations are delayed until $\Phi_i \sim M_{\text{Pl}}/3$. Thus Eq.(4) with $\Phi_i = M_{\text{Pl}}/3$ can be considered as an upper limit on the GW amplitude, i.e. $h^2 \Omega_{\text{gw}}^* \lesssim 10^{-6}$. The same bound follows from [25] with $R_* \lesssim 1/H$ and $\rho_{\text{flat}} \lesssim \rho_{\text{tot}}$ [16].

We confirmed the predictions (3, 4) with intensive lattice simulations. Two different scales have to be kept under control numerically: the mass and the VEV of the condensate, whose ratio is measured by q_i . In our simulations, m defines the natural unit of time while $1/\sqrt{q_i}$ defines the time step required to accurately follow the dynamics. Clearly, we cannot simulate values of q_i as high as 10^{30} . However, we saw above that the linear stage of particle production is independent of q_i , see also [9]. Indeed, the GW spectra that we obtained for different q_i (by varying g^2) were nearly identical, as far as $q\epsilon^4 \gg 1$ was satisfied. We were able to simulate values of q_i ranging from 10^4 to 10^8 , which allowed us to take ϵ ranging from 0.2 to 1. Smaller values of ϵ lead to faster particle production, which could increase the

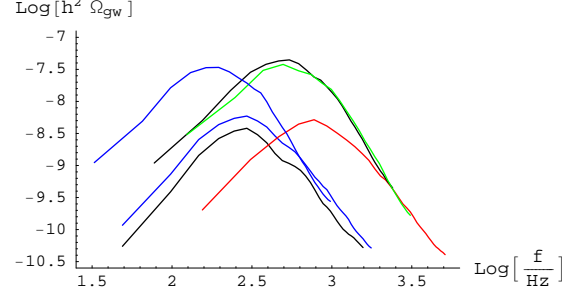


FIG. 3: Lattice results for the GW spectra today, normalized as $h^2 \Omega_{\text{gw}} \times M_{\text{Pl}}^4 / (10 \Phi_i)^4$. Here we did not include the a_i/a_{RD} factors in (2). *Top-left*: RD background, $\Phi_i = 10^{-13} M_{\text{Pl}}$, $m = 122 \text{ GeV}$, $m_\chi = 0$, $q_i = 10^6$, $\epsilon = 0.5$ (blue). *Top-right*: RD background, $\Phi_i = 10^{-1} M_{\text{Pl}}$, $m = 1.22 \text{ TeV}$, $m_\chi = 0$, (i) $q_i = 10^6$, $\epsilon = 0.5$ (black) and (ii) $q_i = 10^8$, $\epsilon = 0.2$ (green). *Bottom-left*: MD background, $\Phi_i = 10^{-13} M_{\text{Pl}}$, $m = 122 \text{ GeV}$, $q_i = 10^6$, (i) $m_\chi = 0$, $\epsilon = 0.5$ (blue) and (ii) $m_\chi = m/5$, $\epsilon = 1$ (black). *Bottom-right*: MD background, $\Phi_i = 10^{-1} M_{\text{Pl}}$, $m = 1.22 \text{ TeV}$, $m_\chi = 0$, $q_i = 10^6$, $\epsilon = 0.5$ (red).

GW amplitude. Increasing the mass of χ has the opposite effect and makes the UV modes relatively more important (which makes this case more difficult to simulate). However, this becomes negligible when m_χ differs significantly from m . This is probably more natural in models with multiple flat directions [10]. We varied g^2 from 10^{-24} to 0.5, Φ_i from $10^{-13} M_{\text{Pl}}$ to $10^{-1} M_{\text{Pl}}$ and m from 100 GeV to 10 TeV, finding excellent agreement with Eqs. (3, 4), see Fig. 3. Note that $\kappa_* \sim 1$ is slightly smaller in a RD background than in a MD one, leading to GW with higher amplitude and smaller frequency at the time of production.

In Fig. 4, we compare our results with the sensitivity of interferometric experiments. We consider the soft masses $m = 100 \text{ GeV}$ and $m = 10 \text{ TeV}$, different initial VEVs Φ_i and different effective temperatures $T_R \propto \rho_{RD}^{1/4}$ when the universe becomes radiation-dominated. If the inflaton decays slowly (perturbatively) and its decay products quickly thermalize, then T_R may correspond to the usual reheat temperature of the universe. In general, however, the universe may become radiation-dominated long before full thermal equilibrium is established, see e.g. [23]. In particular, this means that the standard bound from the thermal over-production of gravitinos does not necessarily apply on T_R . For $T_R \gtrsim 0.2 \sqrt{m M_{\text{Pl}}}$, the universe is RD during the flat direction decay and the GW amplitude is independent of m . If T_R is lower, GW are further diluted and redshifted by the factors in a_i/a_{RD} in Eqs.(2), which depend on m . Assuming a fast transition from MD to RD, we have $a_i/a_{RD} \sim (5 T_R / \sqrt{m M_{\text{Pl}}})^{4/3}$ in that case. We show the results for $T_R = 10^9 \text{ GeV}$ [27] and 10^7 GeV . We see that these GW can be observable for a reasonable range of soft masses and T_R , but this generically requires that some flat directions acquire a large enough initial VEV. A detection would be easier for low m or high T_R , but we should have at least $\Phi_i > 10^{17} \text{ GeV}$.

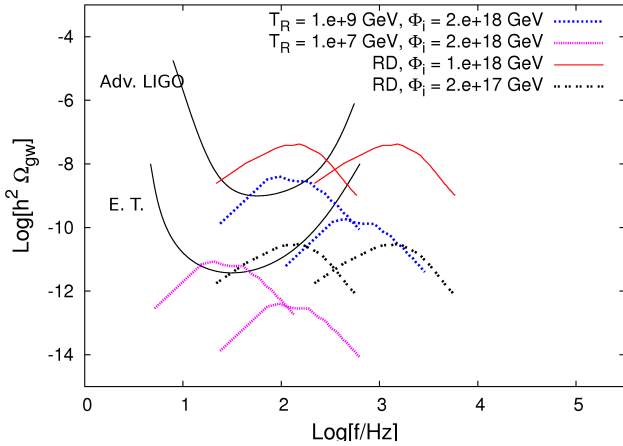


FIG. 4: Sensitivity sketch of Advanced LIGO and Einstein Telescope [1] compared to GW spectra produced in the model (1). In each case, the spectrum at the left is for $m = 100$ GeV and the one at the right for $m = 10$ TeV.

The initial amplitude of flat direction oscillations depends on the lowest order of the non-renormalizable terms that lift the flatness of each direction [3]. A superpotential term like ϕ^n/M^{n-3} (with M the cutoff scale) leads to $\Phi_i \sim (mM^{n-3})^{1/(n-2)}$ [3] when $H \sim m$. In the MSSM, $n = 9$ is the minimal value allowed by gauge invariance for the flattest direction [2]. This gives $\Phi_i \sim 10^{16}$ GeV for M the reduced Planck mass, which would be too small for the GW computed here to be observable by currently proposed experiments. However, the fact that this term is allowed by gauge invariance does clearly not imply that it is indeed present, e.g. it can be forbidden by other symmetries. Higher-order terms lead to larger values of Φ_i and therefore to GW that can be observable. Indeed, initial VEVs can be as high as the

Planck scale [3], e.g. in no-scale supergravity models [8].

To summarize, we performed the first non-linear study of the non-perturbative decay of supersymmetric flat directions with lattice simulations. This explosive process can have important consequences in cosmology, in particular for baryogenesis and reheating after inflation. We showed that it generates a stochastic background of GW that is very sensitive to the initial VEV of flat directions when they start to oscillate, to their soft SUSY-breaking mass and to the time when the universe becomes radiation-dominated. The specific spectral properties that we computed can be used to distinguish these GW from other possible cosmological backgrounds. Of particular interest is that they fall in the Hz-kHz frequency range, where the signal is not expected to be significantly screened by stochastic backgrounds from astrophysical sources. We showed that, even in a simple model, these GW can be observable by upcoming experiments, including Advanced LIGO, if some flat directions acquire a large enough initial VEV. This typically requires the flatness of the potential to be protected up to high order by symmetries, which is model-dependent. Our results strongly motivate the study of more complex models, with multiple flat directions and local gauge invariance [9, 10]. In particular, vector fields are likely to increase the range of parameters leading to an observable signal [15]. Any possibility of direct detection has to be studied in depth, because it would provide crucial informations on both supersymmetry and the very early universe.

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